Mathematics: analysis and approaches
Higher Level
Paper 2

Name

Date: ____________________

2 hours

Instructions to candidates

- Write your name in the box above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- Section A: answer all of Section A in the spaces provided.
- Section B: answer all of Section B on the answer sheets provided. Write your name on each answer sheet and attach them to this examination paper.
- Unless otherwise stated in the question, all numerical answers must be given exactly or correct to three significant figures.
- A clean copy of the **mathematics: analysis and approaches formula booklet** is required for this paper.
- The maximum mark for this examination paper is **[110 marks]**.

exam: 12 pages

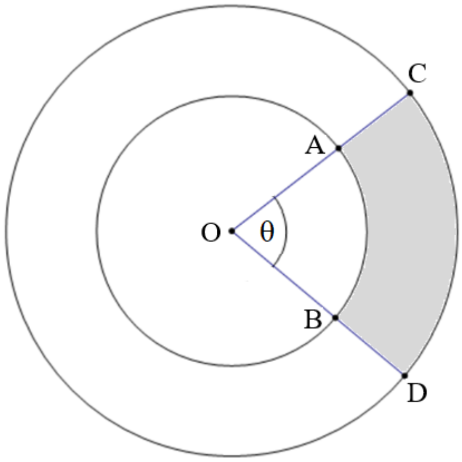
Full marks are not necessarily awarded for a correct answer with no working. Answers must be supported by working and/or explanations. Solutions found from a graphic display calculator should be supported by suitable working. For example, if graphs are used to find a solution, you should sketch these as part of your answer. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. You are therefore advised to show all working.

Section A

Answer **all** questions in the boxes provided. Working may be continued below the lines, if necessary.

1. [Maximum mark: 5]

The diagram below shows two circles which have the same centre O. The smaller circle has a radius of 12 cm and the larger circle has a radius of 20 cm. The two arcs AB and CD have the same central angle θ, where $\theta = 1.3$ radians.
Find the area of the shaded region.



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2. [Maximum mark: 5]

Two lines have the vector equations $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}$.

Find the obtuse angle between the lines.

3. [Maximum mark: 5]

Find the coefficient of the x^3 term in the expansion of $\left(\frac{2}{3}x+3\right)^8$.

4. [Maximum mark: 6]

The table below shows the marks earned on a quiz by a group of students.

Mark	1	2	3	4	5
Number of students	8	7	c	9	1

The median is 3 and the mode is 4 for the set of marks. Find the **three** possible values of c.

5. [Maximum mark: 6]

Consider the complex number $z = \frac{\sqrt{2}}{1-\mathrm{i}} - \mathrm{i}$.

(a) Show that z can be expressed, in the form $x + y\mathrm{i}$, as $z = \frac{\sqrt{2}}{2} + \left(\frac{\sqrt{2}-2}{2}\right)\mathrm{i}$. [2]

(b) (i) Find the **exact** value of the modulus of z.

(ii) Find the argument θ of z, where $-\pi < \theta \leq \pi$. [4]

6. [Maximum mark: 7]

(a) Express $\dfrac{1}{2x^2+7x-4}$ in partial fractions; i.e. as the sum of two fractions. [4]

(b) Given that $\displaystyle\int_1^4 \frac{9}{2x^2+7x-4}\,dx = \ln k$, find the **exact** value of k. [3]

7. [Maximum mark: 6]

(a) Write down the Maclaurin expansion of e^x up to the term in x^4. [1]

(b) Find the Maclaurin expansion of e^{x^2} up to the term in x^4. [2]

(c) Hence, find the Maclaurin expansion of e^{x+x^2} up to the term in x^4. [3]

8. [Maximum mark: 6]

Consider the following system of equations

$$2x + y + 6z = 0$$
$$4x + 3y + 14z = 4$$
$$2x - 2y + (\alpha - 2)z = \beta - 12$$

Find the conditions on α and β for which

(a) the system has no solutions; [2]

(b) the system has only one solution; [2]

(c) the system has an infinite number of solutions. [2]

9. [Maximum mark: 7]

Consider the differential equation $x\frac{\mathrm{d}y}{\mathrm{d}x}+3y=\frac{1}{x}$, $x>0$ such that $y=1$ when $x=1$. Show that the solution to this differential equation is $y=\frac{x^2+1}{2x^3}$.

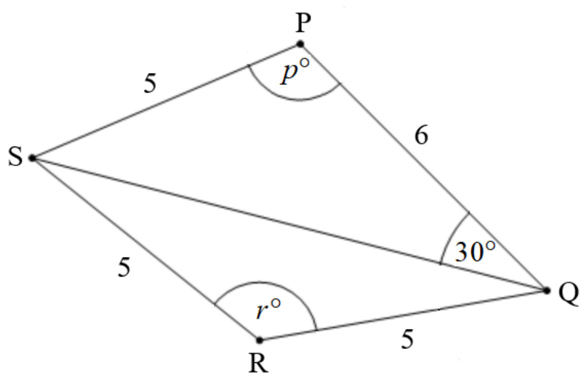
Do **not** write solutions on this page.

Section B

Answer **all** the questions on the answer sheets provided. Please start each question on a new page.

10. [Maximum mark: 15]

The diagram below shows the quadrilateral PQRS. Angle QPS and angle QRS are obtuse.



$\text{PQ} = 6\,\text{cm}$, $\text{QR} = 5\,\text{cm}$, $\text{RS} = 5\,\text{cm}$, $\text{PS} = 5\,\text{cm}$, $\text{P}\hat{\text{Q}}\text{S} = 30°$, $\text{Q}\hat{\text{P}}\text{S} = p°$, $\text{Q}\hat{\text{R}}\text{S} = r°$

(a) Use the sine rule to show that $\text{QS} = 10 \sin p$. [1]

(b) Use the cosine rule in triangle PQS to find another expression for QS. [3]

(c) (i) Hence, find p, giving your answer to two decimal places.

(ii) Find QS. [6]

(d) (i) Find r.

(ii) Hence, or otherwise, find the area of triangle QRS. [5]

Do **not** write solutions on this page.

11. [Maximum mark: 21]

A continuous random variable X has probability density function f defined by

$$f(x)=\begin{cases}\frac{\pi}{3}\sin\left(\frac{\pi}{2}x\right), & 0\le x\le 1\\ mx+b, & 1\le x\le k\\ 0, & \text{otherwise}\end{cases}$$

(a) Given that f is continuous on the interval $0\le x\le k$ and that the graph of f intersects the x-axis at $(k,0)$, show that $k=\frac{\pi+2}{\pi}$. [5]

(b) Find the value of m and the value of b. [3]

(c) Sketch the graph of $y=f(x)$. [2]

(d) Write down the mode of X. [1]

(e) Given that $\int_1^{\frac{\pi+2}{\pi}}\left[x(mx+b)\right]\mathrm{d}x=\frac{3\pi+2}{9\pi}$, find the **exact** value of the mean of X. [7]

(f) Find the value of the median of X. [3]

12. [Maximum mark: 21]

The function g is defined as $g(x)=\mathrm{e}^x+\frac{1}{2\mathrm{e}^x}$, $x\in\mathbb{R}$.

(a) (i) Explain why the inverse function g^{-1} does not exist.

(ii) The line L intersects the curve $y=g(x)$ at points A and B where $x=-1$ at A and $x=1$ at B. Show that the equation of L is $y=\frac{\mathrm{e}^2-1}{4\mathrm{e}}x+\frac{3\mathrm{e}^2+3}{4\mathrm{e}}$.

(iii) Point C is on the curve $y=g(x)$. The line tangent to the curve $y=g(x)$ at C is parallel to L. Find the coordinates of C. [13]

(b) The domain of g is now restricted to $x\ge 0$.

(i) Find an expression for $g^{-1}(x)$.

(ii) Find the volume generated when the region bounded by the curve $y=g(x)$ and the lines $x=0$ and $y=4$ is rotated through an angle of 2π radians about the y-axis. [8]